Mathematics II

(English course)

Second semester, 2012/2013

Exercises (4)

1. For the function $f(x,y,z) = 2x^2y - 3y^2z$, find $f'_{(2,-1,3)}(1,2,-1)$.

2. For each of the following functions, find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and their respective domains.

   (a) $f(x,y) = \begin{cases} x+y, & \text{if } x+y \geq 1, \\ e^{xy}, & \text{if } x+y < 1. \end{cases}$

   (b) $f(x,y) = \begin{cases} \frac{x}{1+y^2}, & \text{if } y \geq -1, \\ -\frac{x}{2y}, & \text{if } y < -1. \end{cases}$

3. Show that the function $f(x,y) = \frac{x-y}{x^2+y^2}$ solves the equation

$$x\frac{\partial f}{\partial x}(x,y) + y\frac{\partial f}{\partial y}(x,y) + f(x,y) = 0 \qquad \forall (x,y) \neq (0,0).$$

4. Consider the function

$$f(x,y) = \begin{cases} \frac{x(1-e^y)^2}{x^2+(1-\cos y)^2}, & \text{for } (x,y) \neq (0,n\pi), \quad n \in \mathbb{Z}, \\ 0, & \text{for } (x,y) = (0,n\pi), \quad n \in \mathbb{Z}. \end{cases}$$

   (a) Find the set of continuity points of f.

   (b) Show that $f'_{(u,v)}(0,0)$ is a well defined real number for each $(u,v) \in \mathbb{R}^2 \setminus \{(0,0)\}$. Compute its expression.

   (c) Which are the values of $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$?

   (d) Is f differentiable at the origin? Why?

5. Consider the function $f(x,y) = \sqrt{|xy|}$.

   Show that the partial derivatives $\frac{\partial f}{\partial x}(0,0)$, $\frac{\partial f}{\partial y}(0,0)$ exist but f is not differentiable at the origin.

6. Find the set of points where the function

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & \text{for } (x,y) \neq (0,0), \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

is differentiable.

7. Consider the function

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{\sqrt{x^2+y^2}}\right), & \text{for } (x,y) \neq (0,0), \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

   (a) Show that f is continuous.
   (b) Find $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.
   (c) Compute the function $\frac{\partial f}{\partial x}$. Show that this function is not continuous.
   (d) Show that f is differentiable at every point.

8. Consider the function

$$f(x,y) = \begin{cases} y\frac{x^2-y^2}{x^2+y^2}, & \text{for } (x,y) \neq (0,0), \\ 0, & \text{for } (x,y) = (0,0) \end{cases}$$

   (a) Show that $f'_{(u,v)}(0,0)$ exists for every $(u,v) \in \mathbb{R}^2 \setminus \{(0,0)\}$.
   (b) Compute the function $\frac{\partial f}{\partial x}$.
   (c) Find the set of all points where $\frac{\partial f}{\partial x}$ is continuous.
   (d) Show that f is not differentiable at the origin.